# Computer Graphics
# Spring 2015 Final

| NAME: |
|---|

| Prob # | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Points | 6 | 9 | 12 | 15 | 16 | 16 | 14 | 12 |
| | | | | | | | | |

Time: 80 Minutes

**NOTES:**

a. **Credit is only given to the correct numerical values.**
b. **All numerical values must be calculated with three digits of accuracy after the decimal point.**
c. **Do not write on the back side of the papers.**

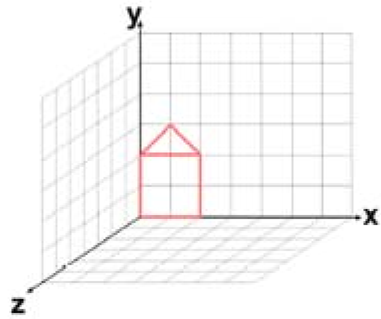
1. In a ray tracing problem, given the equation of a ray $\begin{cases} x(t) = -4t + 12 \\ y(t) = -t + 2 \\ z(t) = -t + 6 \end{cases}$ and a sphere centered at (0,0,4) with radius 4. Find the reflection ray. Intersection point is (4,0,4).

N = ( 4,0,4) – (0,0,4) = (4,0,0)
$N_{normalized}$=(1,0,0)
L=(-4,-1,-1)

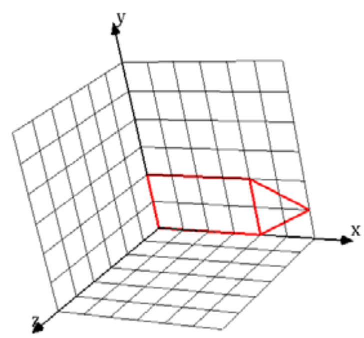
$$L_{Normalized} = \left( \frac{4}{\sqrt{18}}, \frac{1}{\sqrt{18}}, \frac{1}{\sqrt{18}} \right)$$

$$R = 2N(N \cdot L) - L$$
$$R = (-0.9428, 0.2357, 0.2357)$$

2. Given the function *drawHouse()* which creates a wire frame house in the xy-plane as shown in the image below:




Complete the code such that it results in the transformed house as shown in the picture below.




```
glMatrixMode(GL_MODELVIEW)
glLoadIdentity()

glScalef(2,1,1);_________
glRotate(90,0,0,1);______
glRotate(180,1,0,0);_____
drawHouse()
```

3. Equation of a parametric surface is given as

$$x(u,v) = 20\, u^2v^2 + 18\, uv^2 - 4\, v$$
$$y(u,v) = 24\, u^2v^2 - 20\, u^2v + 10u\, v$$
$$z(u,v) = 12\, u^2 + 24\, u\, v - 20\, v$$

Find the normalized normal to this surface at the point corresponding to u=0.25 and v=0.75

Normalized normal to the surface @ u=0.25 and v=0.75 is:__________________

$$\begin{cases} \frac{dx}{du} = 40uv^2 + 18v^2 \\ \frac{dy}{du} = 48uv^2 - 40uv + 10v \\ \frac{dz}{du} = 24u + 24v \end{cases} \text{ and } \begin{cases} \frac{dx}{dv} = 40u^2v + 36uv - 4 \\ \frac{dy}{dv} = 48u^2v - 20u^2 + 10u \\ \frac{dz}{dv} = 24u - 20 \end{cases}$$

$$@u = 0.25, v = 0.75 \begin{cases} \frac{dx}{du} = 15.75 \\ \frac{dy}{du} = 6.7500 \\ \frac{dz}{du} = 24.00 \end{cases} \text{ and } @u = 0.25, v = 0.75 \begin{cases} \frac{dx}{dv} = 4.6250 \\ \frac{dy}{dv} = 3.5000 \\ \frac{dz}{dv} = -14.00 \end{cases}$$

N = [ -178.50 331.50 23.9063 ] Or N = [178.50 -331.50 -23.9063]
N = [ -0.4731 0.8787 0.0634] Or N = [ 0.4731 -0.8787 -0.0634]

4. Consider a parametric surface S(u,v). This surface is linear in the u direction and quadric in the v direction. The parametric equations of the two curves at the two boundaries are given:
Parametric equation of the curve corresponding to u=0: **C0(v) = 3v$^2$ +2v-4**
Parametric equation of the curve corresponding to u=1: **C1(v) = 7v$^2$ +v-2**

a. Find the coefficient matrix C for this surface.(You MUST show the numerical values of the matrix C . No partial grade will be given for an incomplete solution)

$$S(u,v) = \begin{bmatrix} u & 1 \end{bmatrix} \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{bmatrix} \begin{bmatrix} v^2 \\ v \\ 1 \end{bmatrix}$$

$S(u,v) = c_{11}uv^2 + c_{12}uv + c_{13}u + c_{21}v^2 + c_{22}v + c_{23}$

$@v = 0 \quad S(u,0) = c_{13}u + c_{23}$

⇨ $\boldsymbol{c_{13} = 2, c_{23} = -4}$

$@u = 0 \quad S(0,v) = c_{21}v^2 + c_{22}v + c_{23}$

⇨ $\boldsymbol{c_{21} = 3, c_{22} = 2}$

$@u = 1 \quad S(u,1) = c_{11}v^2 + c_{12}v + c_{13} + c_{21}v^2 + c_{22}v + c_{23}$

⇨ $\boldsymbol{c_{11} = 4, c_{12} = -1}$

$$C = \begin{bmatrix} 4 & -1 & 2 \\ 3 & 2 & -4 \end{bmatrix}$$

b. Find the equation of the curve corresponding to u=0.5

$$S(0.5,v) = 5\,v^2 + 1.5v - 3$$

5. The viewing parameters for a perspective projection are given as:

VRP(WC)=(**0,0,0**) VPN(WC)=( **0, 0,1**)
VUP(WC)=(**0,1,0**) PRP (VRC)=(**4,6,8**)

$u_{min}$ (VRC) = **-4** $u_{max}$ (VRC) = **4**
$v_{min}$ (VRC) = **-5** $v_{max}$ (VRC) = **5**
$n_{min}$ (VRC) = **12** $n_{max}$ (VRC) = **18**

Find the sequence of transformations which will transform this viewing volume into a unit cube which is bounded by the planes: x=z ; x=-z ; y=z ; y=-z ; z=zmin ; z=1

a. Find the **Translation matrix** (Matrix #5)
b. Find the **Shear matrix** (Matrix #6)
c. Find the **scale matrices** (Matrix #7 and Matrix #8).
d. Find the **zmin** after all transformations are done.

**Matrix #2: Rx**

| | | | |
|---|---|---|---|
| 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 1 |

**Matrix #1: Translate**

| | | | |
|---|---|---|---|
| 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 1 |

**Matrix #4: Rz**

| | | | |
|---|---|---|---|
| 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 1 |

**Matrix #3: Ry**

| | | | |
|---|---|---|---|
| 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 1 |

**Matrix #6: Shear**

| | | | |
|---|---|---|---|
| 1 | 0 | -0.50 | 0 |
| 0 | 1 | -0.75 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 1 |

**Matrix #5: Translate**

| | | | |
|---|---|---|---|
| 1 | 0 | 0 | -4 |
| 0 | 1 | 0 | -6 |
| 0 | 0 | 1 | -8 |
| 0 | 0 | 0 | 1 |

| | | | |
|---|---|---|---|
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| | | | |
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| | | | |

**Matrix #7: Scale**

| | | | |
|---|---|---|---|
| 0.20 | 0 | 0 | 0 |
| 0 | 0.16 | 0 | 0 |
| 0 | 0 | 0.10 | 0 |
| 0 | 0 | 0 | 1 |

**Zmin = 0.4**

6. Given the triangle ABC in a three dimensional right-handed coordinate system,
A=(2,0,2), B=(5,0,2), C=(4,3,2)
The light source with an intensity of I=10000 is located at (4,4,6) and the viewer (eye) is located at (8,1,5) and $K_a$=0 ; $K_d$=0.4 ; $K_s$=0.6 ; n=2

Given point P(4,1,2) on the triangle ABC:

a. Find the diffuse intensity at point P
Notes:
- Do not use any shading model and ignore $f_{att}$

Vector AB=

$\overrightarrow{AB} = (5,0,2) - (2,0,2) = (3,0,0)$
$\overrightarrow{AC} = (4,3,2) - (2,0,2) = (2,3,0)$
$\vec{N} = \overrightarrow{AB} X \overrightarrow{AC} = (0,0,9)$
$\hat{N} = (\ 0\ 0\ 1)$
$\vec{L} = (4,4,6) - (4,1,2) = (0,3,4)$
$\hat{L} = (0\ ,\ \ 0.6,\ \ 0.8)$
$I_{Diffuse} = I * k_d * (\hat{N} \cdot \hat{L}) = 10000 * 0.4 * 0.8 = 3200$

b. Find the specular intensity at point P from the viewer's point of view
- Do not use any shading model and ignore $f_{att}$

$\vec{V} = (8,1,5) - (4,1,2) = (4,0,3)$
$\hat{V} = (0.8\ \ , 0\ , 0.6)$
$\hat{R} = 2 * \hat{N} * (\hat{N} \cdot \hat{L}) - \hat{L} = (0\ , -0.6, 0.8)$
$\hat{R} \cdot \hat{V} = 0.48$
$I_{Specular} = I * k_s * (\hat{R} \cdot \hat{V})^n = 1382.4$

7. Answer each question.

a) How many triangles are drown if we use eight vertices between glBegin(GL_TRIANGE_STRIP) and glEnd()?
Answer: ____6_____

b) Given the RGB values of a point, R=0.2, G=0.4, B=0.6, Find the CMYK values of that point:

C = 1-R=1-0.2=0.8
M = 1-G=1-0.4=0.6
Y = 1-B=1-0.6=0.4
K = min(C,M,Y)=0.4

c) In a general Bezier bicubic parametric surface with 16 control points, how many of the control points are actually on the surface?
Answer: _____4_____

d) For a given point, the ambient value depends on the location of the observer.
- ☐ True
- ☐ False

e) For a given point, the specular value depends on the location of the observer.
- ☐ True
- ☐ False

f) The specular-reflection coefficient is the same as specular-reflection exponent.
- ☐ True
- ☐ False

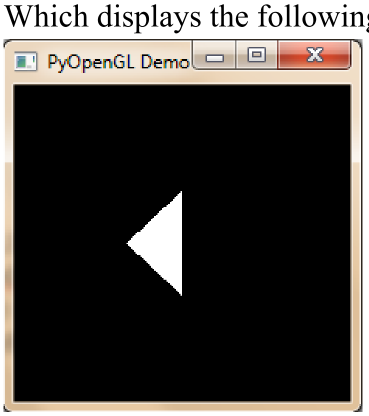
g) The CMY colors are additive primaries.
- ☐ True
- ☐ False

8. Consider the following OpenGL program:

```
from OpenGL.GL import *
from OpenGL.GLU import *
from OpenGL.GLUT import *
def display():
    glClear(GL_COLOR_BUFFER_BIT)
    glBegin(GL_TRIANGLES)
    glColor3f(1,1,1)
    glVertex3f(-1,0,0)
    glVertex3f(1,0,0)
    glVertex3f(0,1,0)
    glEnd()
    glFlush()
    glutSwapBuffers()
glutInit(sys.argv)
glutInitDisplayMode(GLUT_DOUBLE|GLUT_RGB)
glutCreateWindow(b"PyOpenGL Demo")
glutDisplayFunc(display)
glMatrixMode(GL_PROJECTION)
glLoadIdentity()
glRotated(90,0,0,1);

glFrustum(-1,1,-1,1,1,30)
gluLookAt(0,0,3,0,0,0,0,1,0)
glMatrixMode(GL_MODELVIEW)
glLoadIdentity()
glutMainLoop()
```

Which displays the following:



a. If the code in line 23 is replaced with:

```
gluLookAt(0.0, 0.0, 4.0,  0.0,  0.0,  0.0,  0.0,  1.0,  0.0)
```

What happens to the image on the screen? (all other lines stay the same).

☐ The size of the image of the object on the screen get larger
☐ The size of the image of the object on the screen get smaller
☐ The image of the object on the screen moves to the right
☐ The image of the object on the screen moves to the left
☐ The image of the object on the screen rotate clockwise
☐ The image of the object on the screen rotate counter-clockwise
☐ Nothing changes

b. If the code in line 23 is replaced with:

```
gluLookAt(0.0, 0.0, 3.0,  1.0,  0.0,  0.0,  0.0,  1.0,  0.0)
```

What happens to the image on the screen? (all other lines stay the same).

☐ The size of the image of the object on the screen get larger
☐ The size of the image of the object on the screen get smaller
☐ The image of the object on the screen moves down
☐ The image of the object on the screen moves up
☐ The image of the object on the screen rotate clockwise
☐ The image of the object on the screen rotate counter-clockwise
☐ Nothing changes

c. If the code in line 22 is replaced with :

```
gluFrustrum(-1, 1, -1, 1, 0.5, 30)
```

What happens to the image on the screen? (all other lines stay the same).

☐ The size of the image of the object on the screen get larger
☐ The size of the image of the object on the screen get smaller
☐ The image of the object on the screen moves to the right
☐ The image of the object on the screen moves to the left
☐ The image of the object on the screen rotate clockwise
☐ The image of the object on the screen rotate counter-clockwise
☐ Nothing changes

d. If we add this code to line 21:

```
glRotated(15.0, 0.0, 0.0,  1.0)
```

What happens to the image on the screen? (all other lines stay the same).

☐ The size of the image of the object on the screen get larger
☐ The size of the image of the object on the screen get smaller
☐ The image of the object on the screen moves down
☐ The image of the object on the screen moves up
☐ The image of the object on the screen rotate clockwise
☐ The image of the object on the screen rotate counter-clockwise
☐ Nothing changes